A Grammar-based Approach to Invertible Programs

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Background

- Writing a program that is inverse to some other program, such as ...
  - redoing for undoing
  - serialization for deserialization

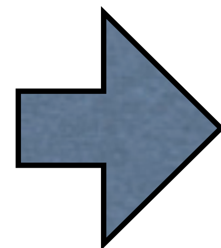
- ... is quite tedious and error-prone.

Program Inversion

Given a program describing function f, find a program describing f^{-1}.

e.g.

Program of
```
snoc(x,b)
 = x++[b]
```



Inv. Prog. of
```
snoc⁻¹(x++[b])
 = (x,b)
```

[Dijkstra: PC78, Gries: 81, Glück&Kawabe: FLOPS04, ...]

This Talk

- Grammar-based inversion
  - uses grammars to approximate programs' evaluation.
    - production vs. evaluation
    - parsing vs. inverse computation
  - Advantages:
    - Clear classification of invertible programs.
    - Less heuristic.
    - Extensible.

Target Programs

- First-Order Functional Programs with call-by-value semantics.
  - e.g.: `snoc`

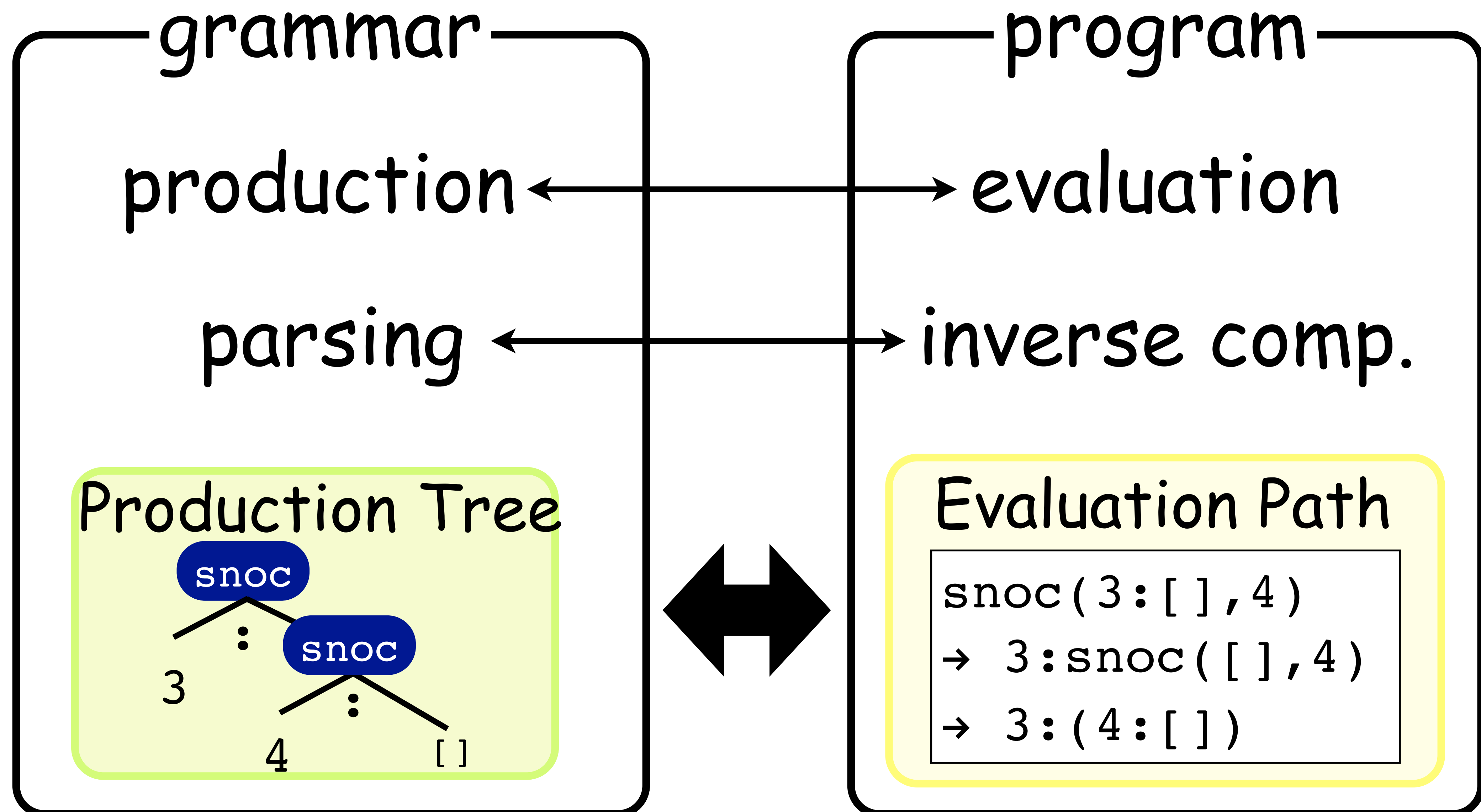
```
snoc( [],b) = b:[]
snoc(a:x,b) = a:snoc(x,b)
```

  - example of evaluation

```
snoc(3:[],4)
→ 3:snoc([],4)
→ 3:(4:[])
```

Idea of Inversion

- Use grammar to approximates program's evaluation.



Generation of Grammar

```
snoc([] ,b) = b:[]
snoc(a:x,b) = a:snoc(x,b)
```

Evaluation

```
snoc(3:[],4)
→ 3:snoc([],4)
→ 3:(4:[])
```

- Observation
  - `snoc(…)` evaluates to `_:snoc(…)` or `_:[]`

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snoc([] ,b) = b:[]
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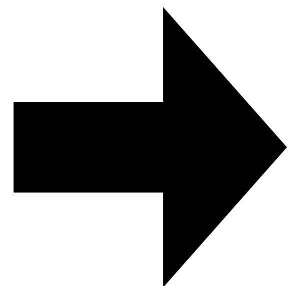

Grammar

```
snoc →█:[]
snoc →█:snoc
```

Grammar represents func.-call traces.

Inverse Comp. by Parsing

```
snoc([] ,b) = b:[]
snoc(a:x,b) = a:snoc(x,b)
```

Evaluation

```
snoc(3:[],4)
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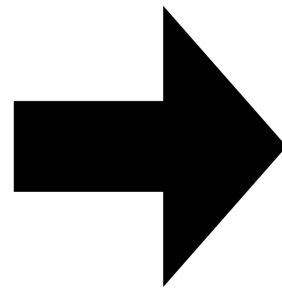

Grammar

```
snoc
  → █:[]
snoc
  → █:snoc
```

$\texttt{snoc}^{-1}$

3:4:[]

Inverse Comp. by Parsing

```
snoc([] ,b) = b:[]
snoc(a:x,b) = a:snoc(x,b)
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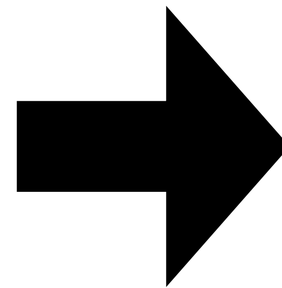

Grammar

```
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snoc
  → █:snoc
```

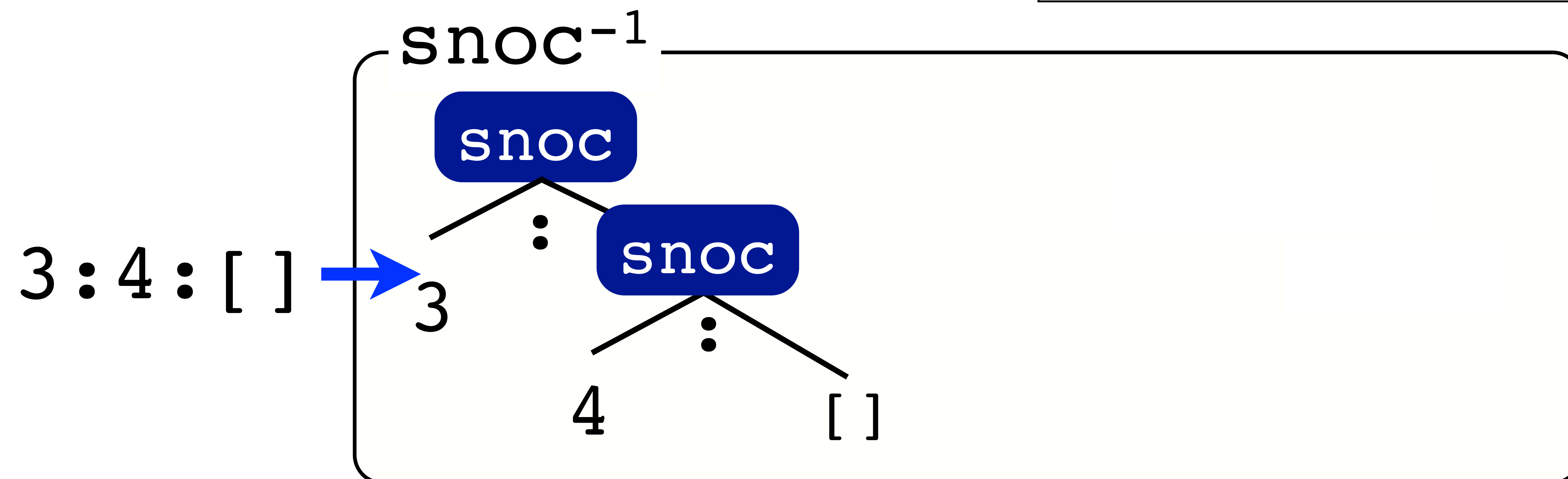

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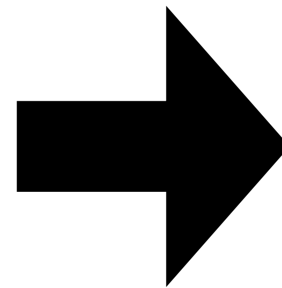

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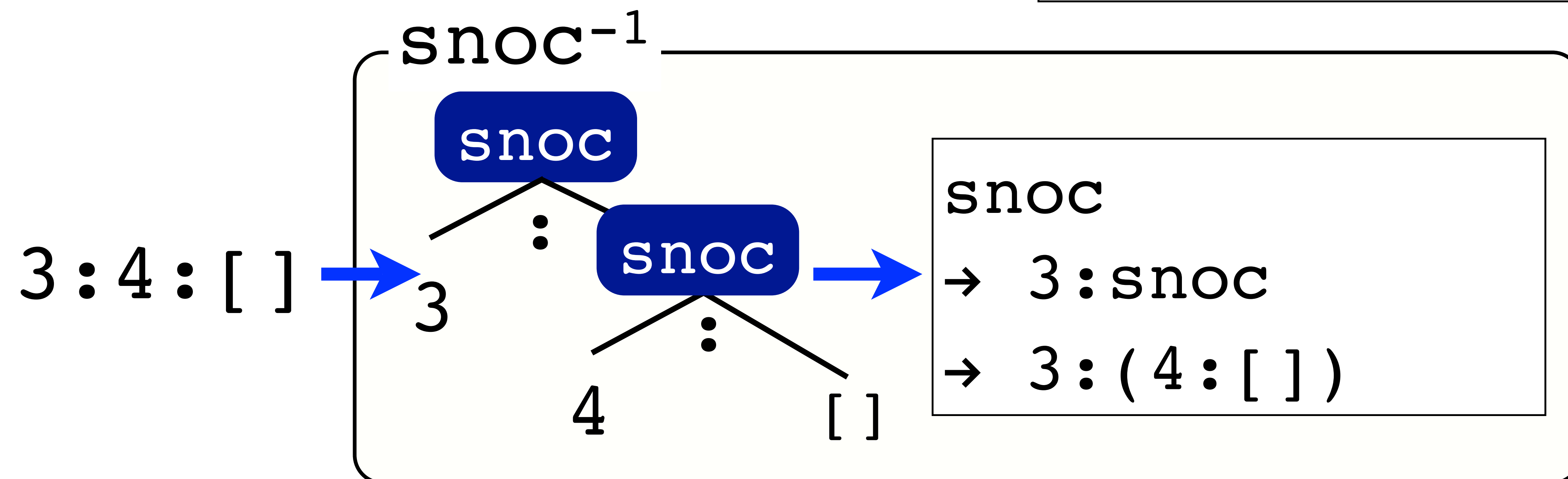

Inverse Comp. by Parsing

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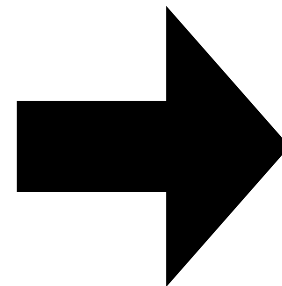

Grammar

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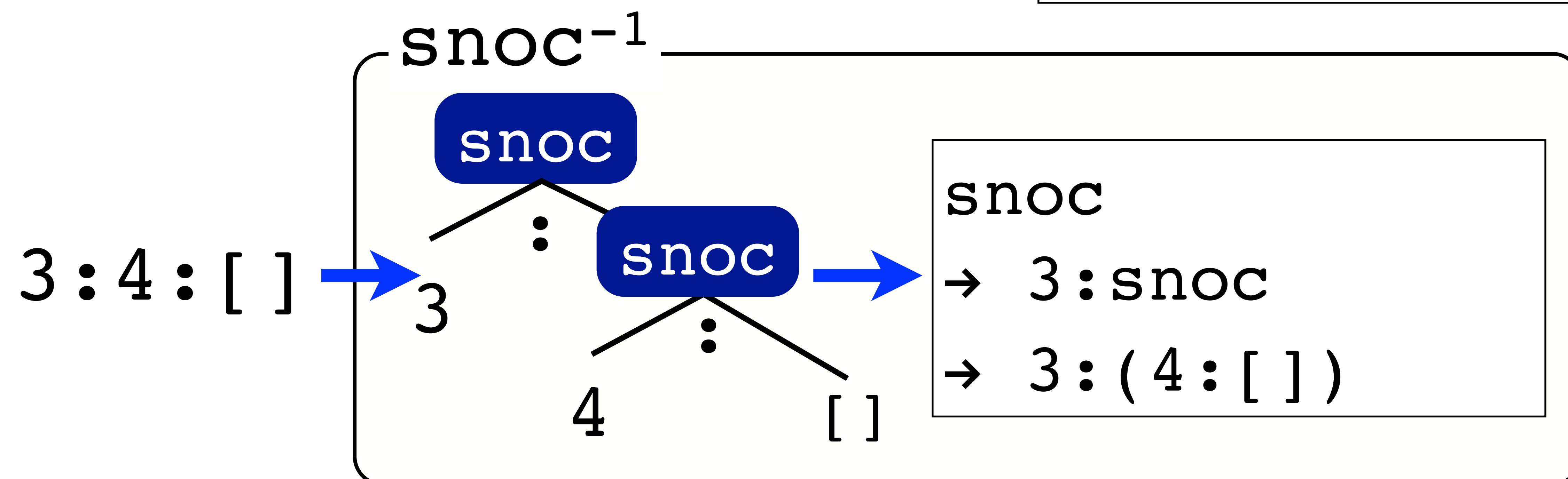

Inverse Comp. by Parsing

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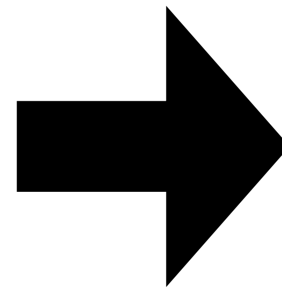

Grammar

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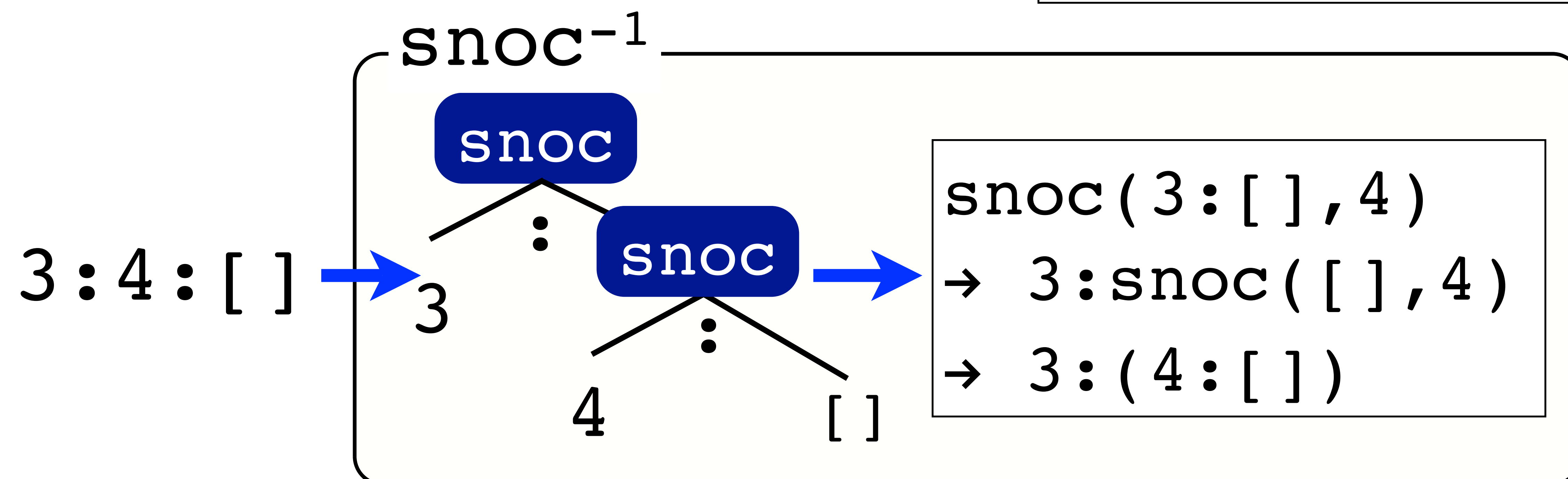

Inverse Comp. by Parsing

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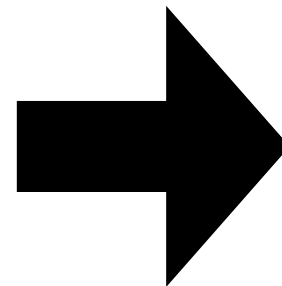
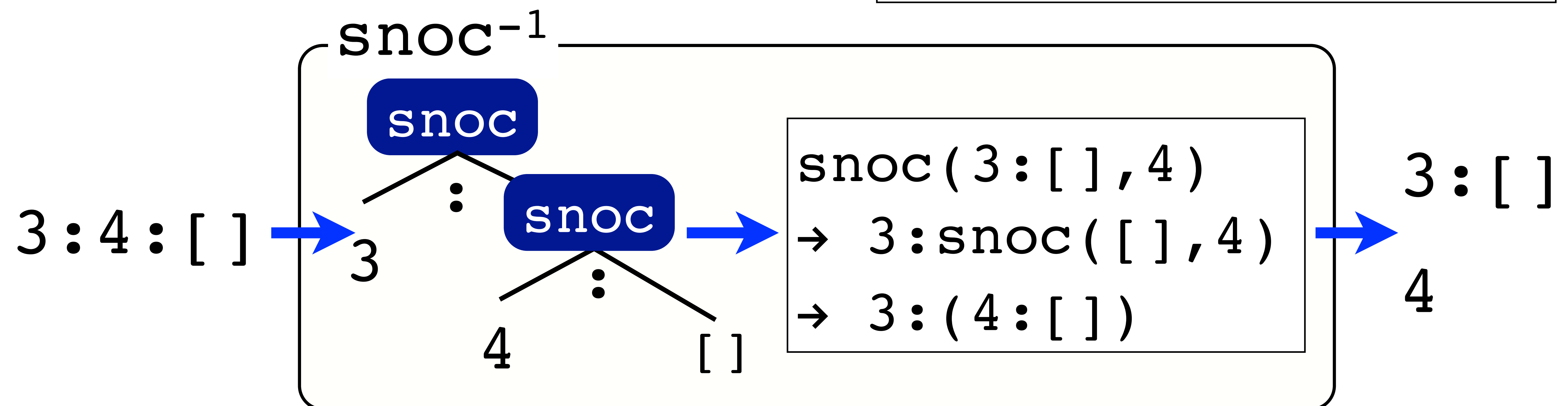


Grammar

```
snoc( [],b)
 → b:[]
snoc(a:x,b)
 → a:snoc(x,b)
```



Whole Inversion System

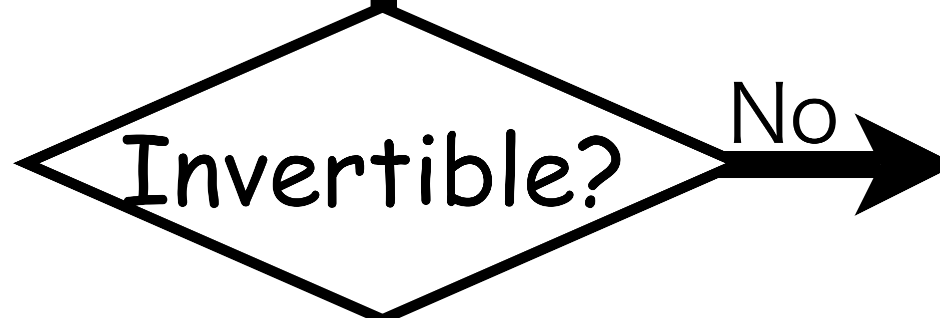
Input Program

`snoc(x,b) = …`

generate!

Grammar

`snoc → ■:snoc …`



Fail

Yes

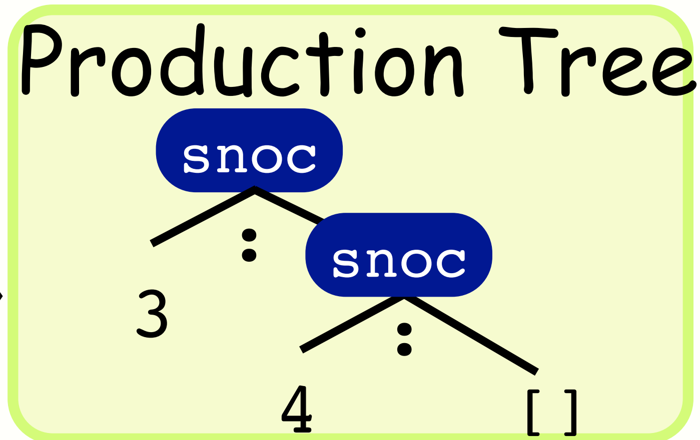
generate!

Output Inverse

`snoc⁻¹(r)=…`

input of inverse

`3:4:[]`



reconstruct

Evaluation Path

```
snoc(3:[],4)
→ 3:snoc([],4)
→ 3:(4:[])
```

output of inverse

`(3:[],4)`

`snoc → ■:snoc …`

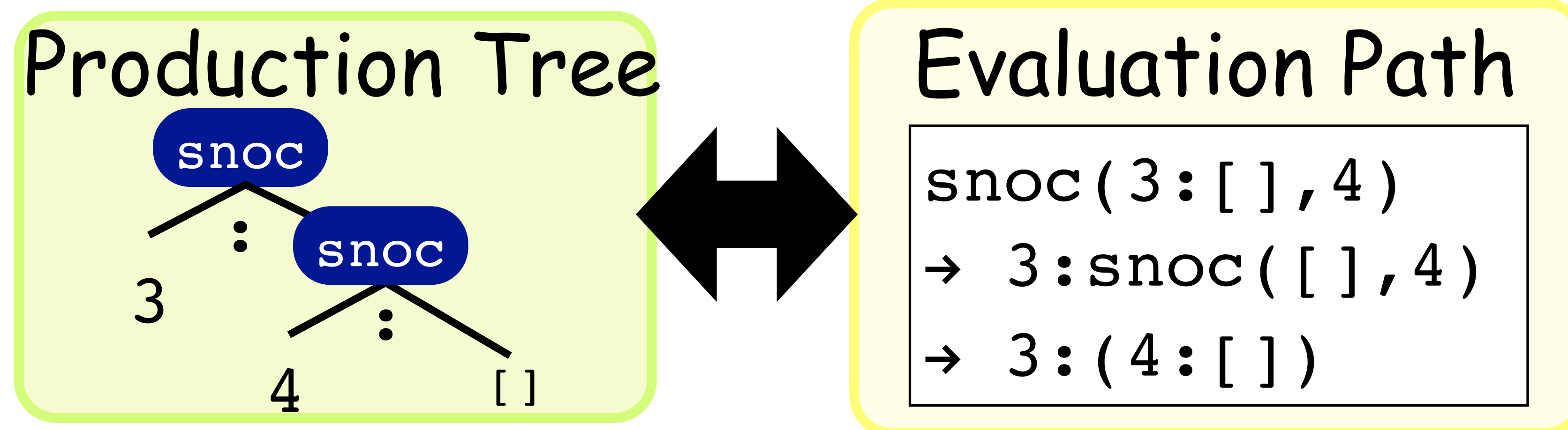
Correctness

Theorem

If a program is non-erasing and the derived grammar is unambiguous, then the derived inverse is actually inverse.

Proof.

Bijection between production&evaluation



Efficiency

- The complexity of a derived inverse depends on the complexity of parsing.

Remark.

If a program is linear and treeless, the derived inverse runs in O(n) time, where n is the size of the input of the inverse.

- Treeless [Wadler: TCS90]
  - Every function-call has the form: $f(x_1,\ldots,x_n)$
  - e.g.: snoc

Advantages

- Classification of invertible programs
- Less heuristic
- Extensible

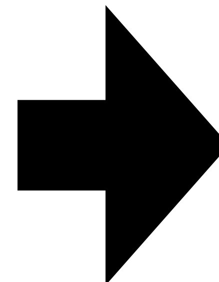
1: Classification

- How hard it is to invert a program?

  ⇐ by grammar's hierarchy.

by Regular Tree Grammar

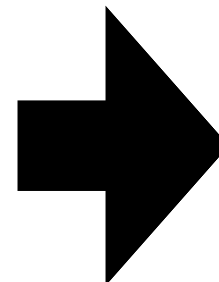
```
snoc( [],b) = b:[]
snoc(a:x,b) = a:snoc(x,b)
```



```
snoc →█:[]
snoc →█:snoc
```

by (IO) Context-Free Tree Grammar
[Engelfriet&Schmidt: J.Compt.Syst.Sci77]

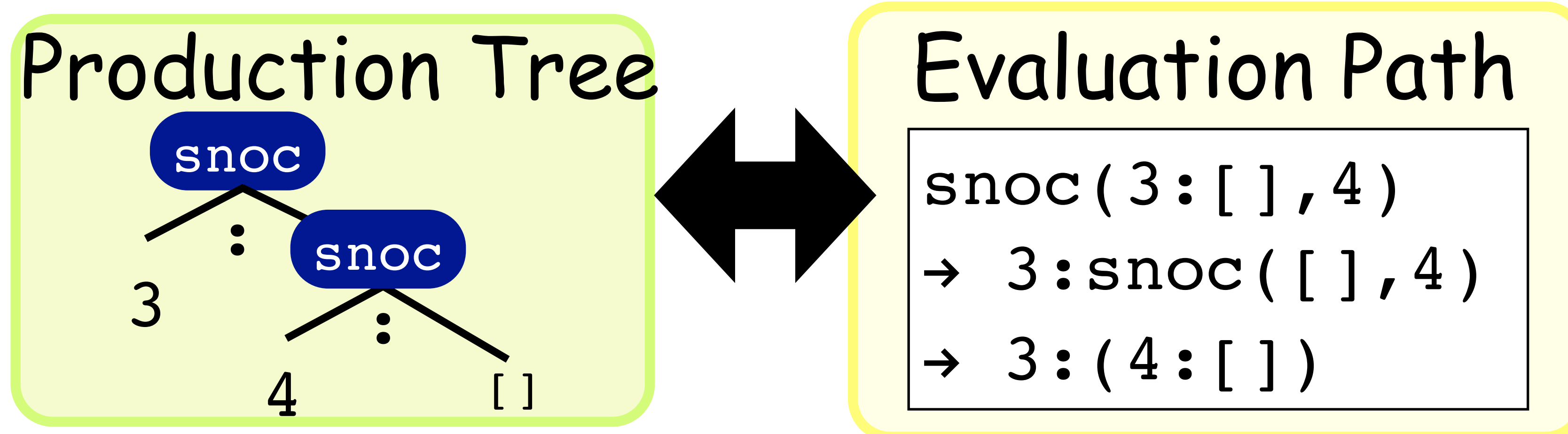
```
reverse(xs) = rev(xs,[])
rev([],y)   = y
rev(a:x,y) = rev(x,a:y)
```



```
reverse → rev([])
rev(y) → y
rev(y) → rev(█:[])
```

2: Less heuristic

- Grammar-based inversion is less heuristic.
  - based on the correspondence between evaluation/production.



3: Extensible

- Function compositions

```
snoc2(x,b,c) = snoc(snoc(x,b),c)
```

  - Reparse the result of outer "snoc".
- Grammars
  - Regular tree grammars
    - `snoc`, and treeless programs
  - IO Context-free tree grammar
    - `reverse`, and

      LMTTs [Engelfriet&Vogler: J.Compt.Syst.Sci85]

Short Summary

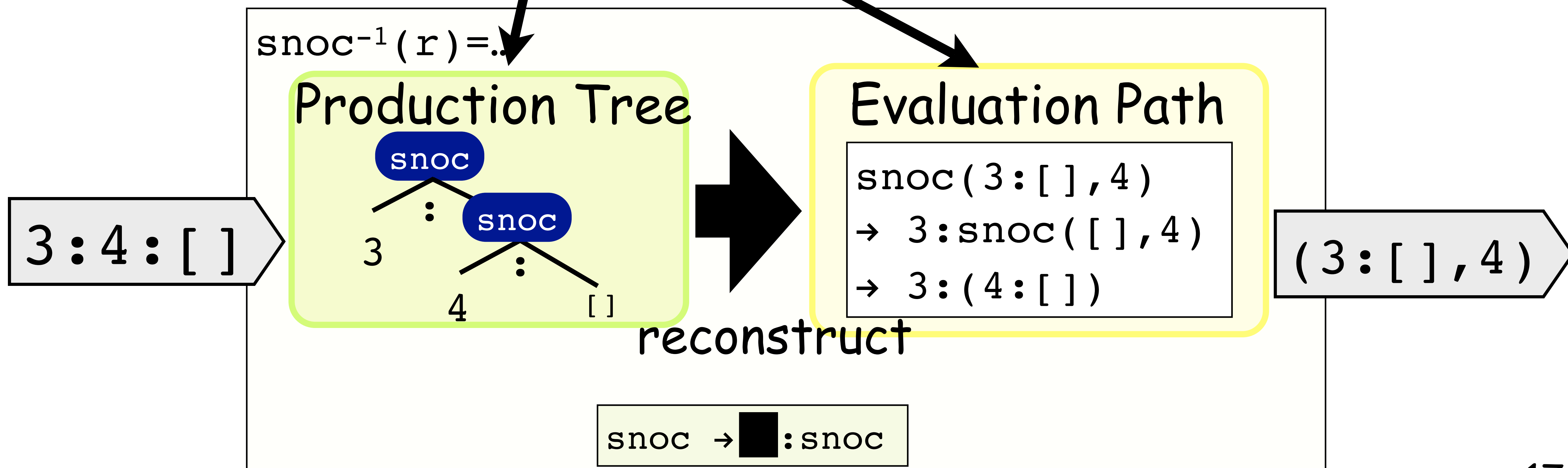
- Grammar-based Inversion
  - approximates program evaluation by grammar's production.
  - characterizes program's invertibility by grammar's unambiguity.
  - is based on bidirectional conversion of evaluation tree/production tree.

```
snoc( [],b) = b:[]
snoc(a:y,b) = a:snoc(y,b)
```

```
snoc → █:[]
snoc → █:snoc
```

Is it efficient?

- A derived inverse seems to entail a lot of overhead due to large amount of intermediate data.



Experiments

prototype impl.: http://www.ipl.t.u-tokyo.ac.jp/~kztk/PaI/

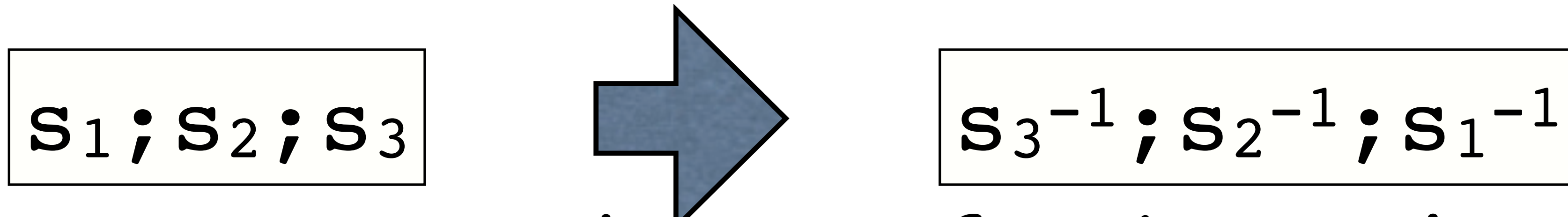
- Comparison to handwritten inverses.
  - to know overhead caused by "parsing".

Program	#input	handwritten	derived	speed ratio
snoc	8M	0.67s	0.95s	1.4
dbl	10M	0.11s	0.23s	2.1
zip	8M	0.28s	0.70s	2.5
runlength	9M	0.33s	0.76s	2.3

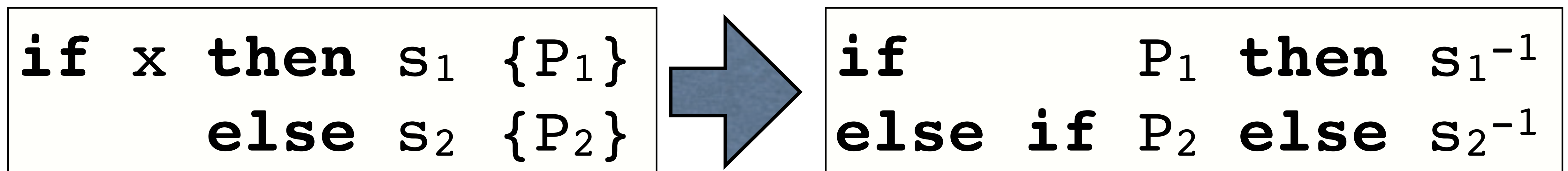
Acceptable overhead!

Related Work

- Many of existing work
  - revert execution order



  - use post conditions for branches



- Our work is based on production/evaluation bijection.

Closely Related Work

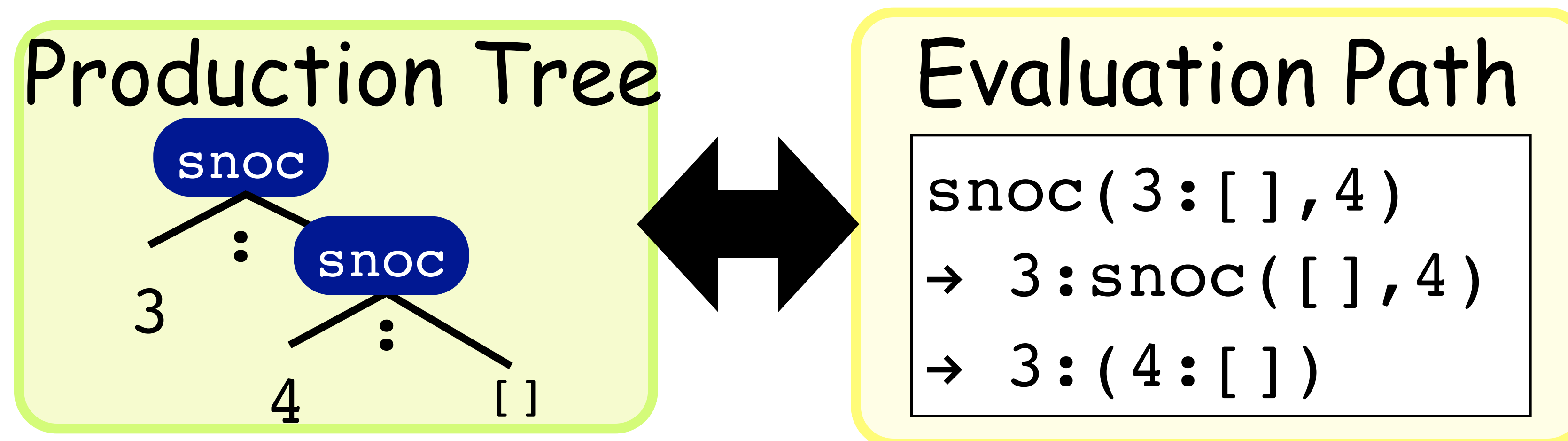
- [Glück&Kawabe: FLOPS04]
  - LR-parsing techniques to determines inverse programs
- [Yellin: ICSE88]
  - Program inversion of some class of AG

- Our work is a generalization and reformulation of these works.

Conclusion

prototype impl.: http://www.ipl.t.u-tokyo.ac.jp/~kztk/PaI/

- Grammar-based Inversion
  - ... is based on bijective mapping between production and evaluation.



  - ... classifies programs by unambiguous grammars requires in inversion.
    - unambiguity (+α) implies invertivility!

Future Work

- More investigation on the idea
  - $\exists x. f^{-1}(y) = x \Leftrightarrow y \in Range(f)$
- ... for right-inverses.

  - Applications
    - Compression & Decompression
    - Testing: $\forall x. P(x) \rightarrow Q(x)$ by using P^{-1}
      [Runciman et al.: Haskell08, Fischer: D.Thesis, ...]
    - and so on.

Programs

- First-Order Functional Programs with call-by-value semantics.
  - e.g.: `snoc`

```
snoc( [],b) = b:[]
snoc(a:x,b) = a:snoc(x,b)
```

  - example of evaluation

```
snoc(3:[],4)
→ 3:snoc([],4)
→ 3:(4:[])
```

(3:4:[] || [3,4])

Problem of Existing Methods

- It is unclear which programs are invertible

	dbl	snoc	reverse
[Glück&Kawebe: SIGPLAN Notices05]	OK	NG	NG
[Mogensen: GPCE 05]	OK	NG	NG
[Nishida&Sakai: ENTCS 09]	OK	OK	NG
[Glück&Kawebe: FLOPS 04]			

```
dbl(Z) = Z
dbl(S(x)) = S(S(dbl(x)))
```

```
reverse(x) = rev(x,[])
rev(a:x,r) = rev(x,a:r)
rev( [],r) = r
```

```
snoc( [],b) = b:[]
snoc(a:x,b) = a:snoc(x,b)
```

Questions we Answer

- Classification of invertible programs.
  - Which ones are essentially easy to invert, and which ones are difficult?
  - How efficient inverses can be derived for the programs?

```
dbl(Z) = Z
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snoc( [],b) = b:[]
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```

Classification Result

Every Invertible function

By Context-free Tree Grammar

`reverse`, ...

By Regular Tree Grammar

`snoc`, ...

(top-down deterministic)

`dbl`, ...

Existing Methods

	dbl	snoc	reverse
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Very Easy

```
dbl(Z) = Z
dbl(S(x)) = S(S(dbl(x)))
```

a bit Hard

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reverse(x) = rev(x,[])
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Easy

```
snoc( [],b) = b:[]
snoc(a:x,b) = a:snoc(x,b)
```

Issues of Program Inversion

- For efficient inverses, an inversion method often targets only a restricted class of programs.

- ... but what class should we target?

Existing Method targets ...

	dbl	snoc	reverse
[Glück&Kawebe: SIGPLAN Notices05]	OK	NG	NG
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Is this result essential?

- intrinsic to the program?
- or inadequacy on the method?

Question

- Classification of invertible programs.
  - Which ones are essentially easy to invert, and which ones are difficult?
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Our Approach

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snoc( [],b) = b:[]
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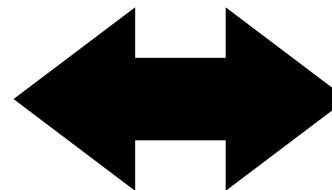

```
snoc → ■:[]
snoc → ■:snoc
```

Our Answer to the Question

- Classification of invertible programs.
  - Which ones are essentially easy to invert, and which ones are difficult?

  ⬅ By how complex grammar is used

  - How efficient inverses can be derived for the programs?

  ⬅ By the computational complexity of parsing with the grammar

Classification Result

Every Invertible function

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By Regular Tree Grammar

`snoc`, ...

(top-down deterministic)

`dbl`, ...

Idea of Grammar-Based Inversion